

# Dynamic FE analysis of a slope using seismic time-history loading

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**Abstract**— FE analysis for predicting earthquake-induced landslide displacements provides a widely used way to predict the displacements affecting a sliding mass subject to earthquake loading. In this model, seismic slope stability is measured in terms of critical acceleration, permanent displacement and safety factor. The triggering seismic forces are investigated in terms of acceleration records of Boumerdes earthquake. The method is based on the dynamic calibration strategy (for good results in adjustments the frequency domain) of relations having the general form of an attenuation law that relates the energy of the seismic forces to the dynamic shear resistances of the sliding mass to propagate the expected landslide displacements. The permanent displacement resulting from this analysis is compared to Newmark's model for predicting earthquake-induced landslide displacements to demonstrate results of this study. \*CRITICAL: Do Not Use Symbols, Special Characters, or Math in Paper Title or Abstract. (Abstract)

**Index Terms**— FE modelling; permanent displacement; dynamic calibration

## I. INTRODUCTION

The usual design approaches to analyses the seismic slope stability typically refer to two classes of methods: pseudo-static, in which the seismic action is represented by an “equivalent acceleration” used in a conventional limit equilibrium slope stability analysis (fig 1); and The displacement-based methods for ascertaining the stability of slopes during earthquakes which are based on the estimation of the extent of displacement of a mass of soil along a failure surface by using the scheme of a rigid block sliding on an inclined plane (Fig. 2).

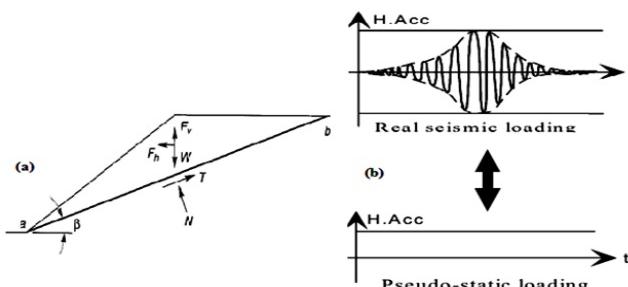


Fig. 1. Principe de la simplification dans l'analyse P-S.

In both two classes of methods, soil deformability and coupling between dynamic response of the system and the frequency content of the seismic motion are not considered.

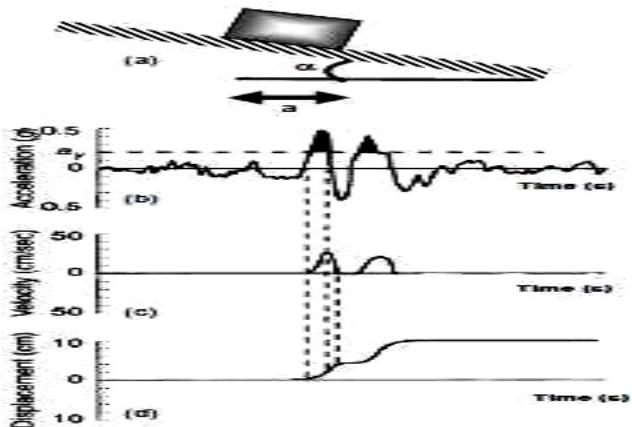


Fig. 2. Potential sliding block under earthquake shaking.

Since helpfulness slope after an earthquake is controlled by its deformations, the analyzes which provide for displacement in the slope provide a more useful indication for calculating seismic stability. Newmark has proposed the basic elements of a procedure to assess potential displacements due to seismic waves of a dike or dam rather than the minimum safety factor. The failure mechanism is defined as the factor limit (threshold)  $k_y$  and critical acceleration limit (threshold)  $a_y = k_y g$  corresponding to the minimum flow resistance and which are to be determined first. The above threshold pulse acceleration  $a_y$  are properly integrated in time to estimate any movement of the sliding mass potential.

The usual design approaches to analyses the seismic slope stability typically refer to two classes of methods:

## II. PERMANENT DISPLACEMENT CONCEPTS

### A. Influence of the critical acceleration on the displacement of the slope:

The permanent displacement of the slope depends on the ratio between the critical acceleration and maximum acceleration. Obviously, the sliding block model predicts zero permanent displacement of the slope if the accelerations induced by the earthquake never exceed the critical acceleration ( $a_y / a_{max} \geq 1$ ) as shown in Fig.3 (a).

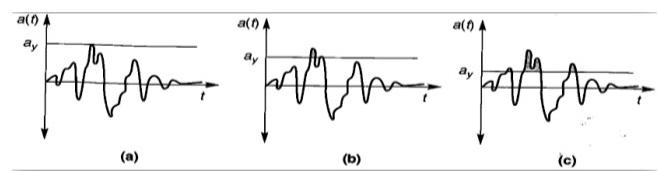


Fig. 3. (a) If  $a_y \geq a_{max}$ , no permanent displacement will occur; (b) and (c) the permanent of the slope is not

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### B. Concept of permanent displacement on slopes

Newmark in 1965 first developed a simple model to estimate the permanent displacement of a slope. Newmark results are reproduced in Fig.4. The normalized displacement ( $d_s$ ) can be converted into actual displacement ( $d$ );

$$d = d_s \frac{0.86 v_m^2}{k_m} \quad (1)$$

$d_s$  is the normalized displacement [m];  $v_m$  is the maximum ground speed of the earthquake in [m / s];  $k_m$  is the maximum horizontal ground acceleration expressed as a fraction of the constant of gravity (g).

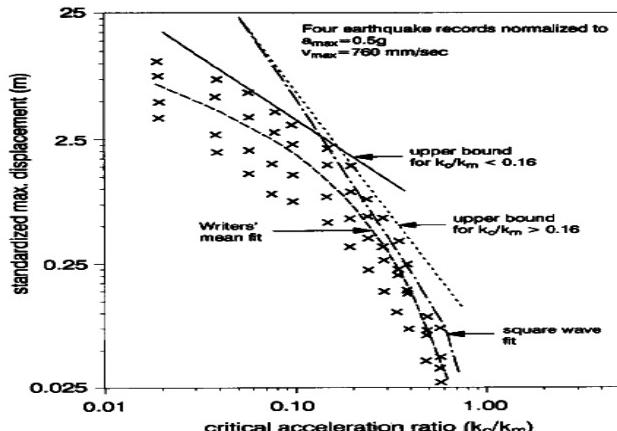


Fig. 4. The standard displacement of Newmark for standardized earthquakes.

Franklin Chang applied the theory Newmark and they built 169 records of strong movements for 27 and 10 earthquakes accelerograms. Normalized displacements were calculated for each record according to the method of Newmark. The large number of records analyzed by Franklin Chang gave results in a wide range of movements. Thus the ratio between the standard displacement and accelerating critical report for selected groups of data was expressed in terms of upper limit for each group (fig.5).

Each normalized curve of displacement shown in Fig.5 gives an assessment of the maximum possible standardized displacement of a sliding mass when the critical acceleration ratio is known. The normalized displacement can be converted into actual displacement.

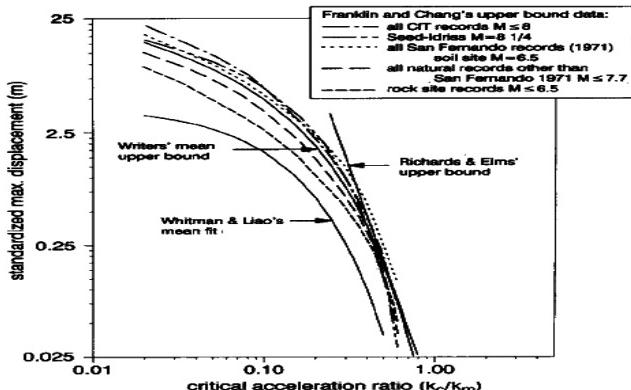


Fig. 5. Franklin Chang standard displacement and proposed approximations.

Other researchers have empirically (deterministic or probabilistic methods) assessed the risk of landslides triggered by the earthquake using movement of Newmark with appropriate formulas. Here we cited some methods that will be used in the digital application.

- Methods using the maximum acceleration and maximum velocity:

#### Newmark:

$$d = 9.2 \frac{v_m^2}{k_m g} \exp(-5.87 \frac{k_c}{k_m}) \left( \frac{k_c}{k_m} \right)^{-0.49} \quad (2)$$

**Whitman and Liao** performed a regression analysis on Franklin Chang standardized data and suggested the following formula to estimate the average permanent displacement:

$$d = 37 \frac{v_m^2}{k_m g} \exp(-9.4 \frac{k_c}{k_m}) \quad (3)$$

**A. authors:** some authors performed a regression analysis on the entire range of normalized displacement data provided by Franklin Chang.

$$d = 35 \frac{v_m^2}{k_m g} \exp(-6.91 \frac{k_c}{k_m}) \left( \frac{k_c}{k_m} \right)^{-0.38} \quad (4)$$

- Methods using the maximum acceleration and intensity:

**Ambraseys and Menu:** they applied Newmark's theory to integrate a number of seismic records and conducted regression analyzes performed on several variables on the permanent displacement of calculated data with several seismic characteristic parameters as independent variables.

$$\log(d) = 0.9 + \log\left[1 - \frac{k_c}{k_m}\right]^{2.53} \left(\frac{k_c}{k_m}\right)^{-1.09} + 0.3t \quad (6)$$

$t$  is a function of the chosen confidence level; Distance from the source less than 45% of the source dimensions and magnitude earthquake in the interval  $6.4 \leq M \leq 7.7$  and  $0.1 \leq a_y / a_{max} \leq 0.9$ ,

**Jibson et al, (1998):** correlated the sliding block movement with the intensity of Arias

$$\log u = 1.521 \log I_a - 1.993 \log a_y - 1.546 \pm 0.375 \quad (7)$$

The Arias intensity can be estimated by the following reports:

Wilson, 1993 (Argyroudis, 2006) :

$$\log I_a [\text{m/s}] = M_w - 2\log R - 3.990 + 0.365(1-P) \quad (8)$$

Ou Arias, 1970 :

$$I_a = \frac{\Pi}{2g} \int_0^{T_d} (a(t))^2 dt \quad (9)$$

where  $g$  is the acceleration due to gravity,  $a(t)$  is the recorded acceleration time-history and  $T_d$  is the duration of the ground motion.

### C. Critical permanent displacement

The critical displacement is defined as the post-seismic displacement at which cracking occurred, shear resistance along the sliding surfaces approaching residual values, and a general breakdown of the sliding mass can occur. The critical displacement depends on the rheological behavior of the sliding masses; the masses that show brittle behavior have a lower critical displacement than masses whose ductility suits larger deformations before sliding. Simplified procedures proposed in the literatures define an equivalent seismic coefficient =  $k_{eq} f_{eq} (a_{max}/g)$  to calibrate the pseudo-static method to a particular level of slope performance (Table 1):

TABLE I. DEVICES FOR SIMPLIFIED PROCEDURES FOR THE SEISMIC STABILITY OF SLOPE (KRAMER AND STEWART, 2004).

Seed (1979)	H-G et F. (1984)	Bray et al. (1998)	Stewart à al. (2003)
barrages	barrages en terre	embankment	Urban slope
M	6.5 - 8.25	3.8 - 7.7 (6.6)	8
$d_y$	100 cm	100 cm	15 - 30 cm
$k_e$	-	0.5	0.75
$\eta$	0.1, 0.15	0.5 $a_g/g$	0.5 $a_g/g$
$F_s$	1.15	1.0	1.0

H-G ET F. (1984) : HYNES-GRIFFON ET FRANKLIN (1984)  
M (MAGNITUDE);  $d_y$  (LIMIT MOVE);  $k_{eq}$  (EQUIVALENT SEISMIC COEFFICIENT);  $F_s$  (SAFETY FACTOR)

### III. FINIT ELEMENT ANALYSIS

The stress-strain seismic slope stability analysis is always performed using the dynamic finite element analyzes. In such analyzes permanent deformations induced by the earthquake in each element of the finite element mesh are built for permanent deformation of the slope. Permanent deformations in the individual elements can be estimated in different ways:

- Reduction approaches for potential stress and rigidity consider permanent deformations using the results of laboratory tested to determine the stiffness of soils susceptible to seismic loading.
- Nonlinear analysis using the non-linear stress-strain behavior of the soil to determine the development of permanent deformation during an earthquake.

Today, because of the progress of powerful computers, stability calculations are also possible and have become rather simple for the development of finite element codes (EF). Such codes are widely applied in the geotechnical field and included in assessments of slope stability

It is usually very time consuming to carry out analysis in the time domain for Any finite element programs, as Digitised Each earthquake load Will Be Treated as separate static load and the program must process the matrix of equations For the entire mesh. Because of this, the trade finite element programs usually adopt frequency domain analysis solution to earthquake loads. Example of the programs That uses this method is PLAXIS;

### IV. DYNAMIC SLOPE STABILITY ANALYSIS USING FE MODELLING

The general algorithm for solving nonlinear equations of the PLAXIS code is done in an iterative process to determine at each time step of calculating fields of displacements, velocities and accelerations corresponding to the applied loads; balance the resulting rapidly. Fig.6 shows the steps of a dynamic calculation.

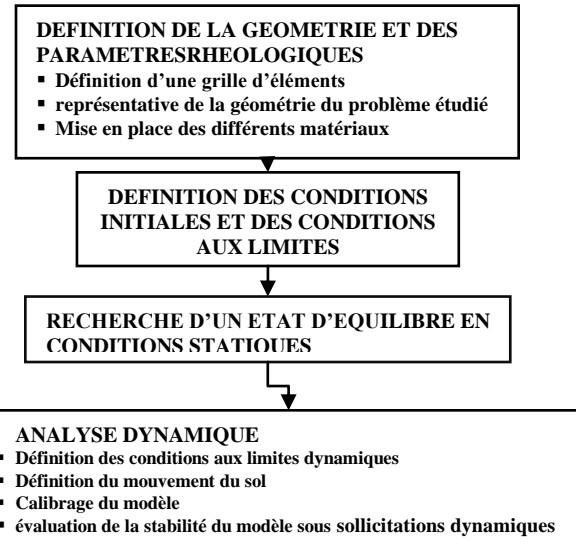


Fig. 6. The general algorithm for solving nonlinear equations of the PLAXIS

In order to validate the method chosen for our work, a slope stability study was conducted on an idealized model of paying mere geological and topographical configuration

#### D. Slope and ground moution characteristics

The geometry of the studied model has a uniform slope length of 400 m, slope height of 20 m and inclination  $\alpha = 22^\circ$  (Fig.7).

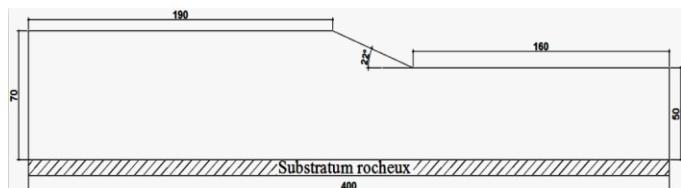


Fig. 7. Geologic and topographic configuration of the studied slope.

The physical and mechanical properties of the material used are:

$$\gamma = 19 \text{ kN/m}^3 ; \nu = 0.30 ; E = 2.78 \times 10^5 \text{ kPa} ; \phi = 30^\circ$$

$$V_s = 234.7 \text{ m/s} ; C = 15 \text{ kPa} ; G = 1.06 \times 10^5 \text{ kPa} ; \psi = 0^\circ$$

To assume the stiffness increases with depth a linear law is supposed to describe the evolution of the shear modulus G and shear wave velocity (Fig.8) with depth z (power exponent m=1/2) :

$$G = G_0 (1 + az)^{2m} = G_0 (1 + az) \quad (10)$$

$$V_s = V_{s0} (1 + az)^m = G_0 (1 + az)^{1/2} \quad (11)$$

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$V_{S0}$  where is the shear wave velocity at the free surface,  $a$  is a coefficient that represents the level of heterogeneity

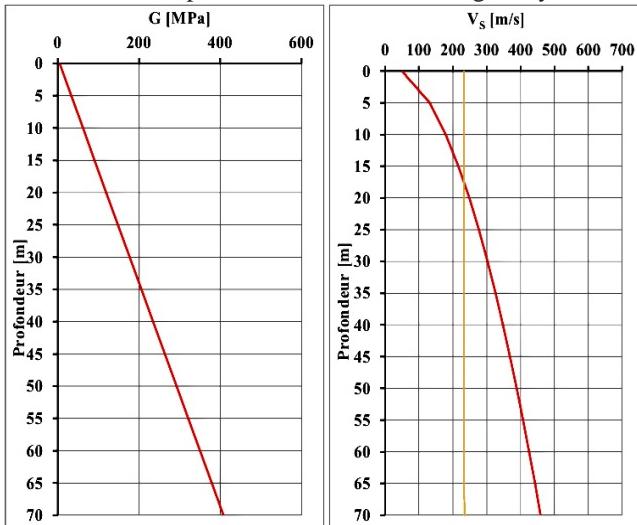


Fig. 8. Profiles of shear wave velocity vs and shear modulus G.

The load signal used is the NS component of the accelerometer recording the Keddara station (Figure 9), some characteristic parameters are summarized in Table 2.

TABLE II. SOME CHARACTERISTIC PARAMETERS OF SOIL MOVEMENT.

Characteristic	Keddara-NS
Maximum acceleration $a_{max}$ (PGA)	331.55 cm/s <sup>2</sup>
Maximum velocity $V_{max}$ (PGV)	17.893 cm/s
Predominant frequency $f$	4.126 Hz
Arias Intensity $I_a$	66.9 cm/s

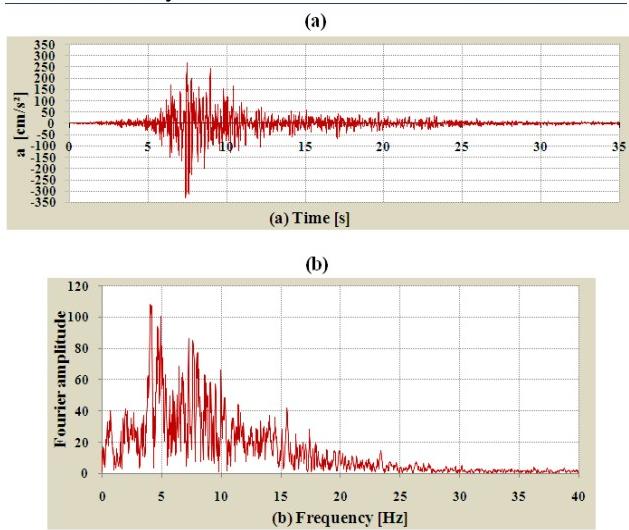


Fig. 9. earthquake data, (a) the accelerogram (b) Fourier spectrum of the Keddara station.

### E. Slope stability analysis in the static conditions

In Plaxis, the safety factor can be made by a process called Phi-c reduction by reducing soil strength parameters

The FS-STAT safety factor obtained for a purely static analysis (in pouring weight only) is  $F_{S-STAT} = 1.13$  which is represented in terms of computational steps "calculation step" in Fig.7 and 8.

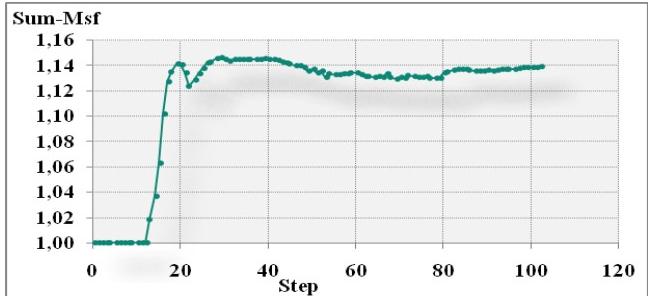


Fig. 10. Safty factor for the static slope stability conditions

The obtained value of FS-STAT indicates an uncertain safety.

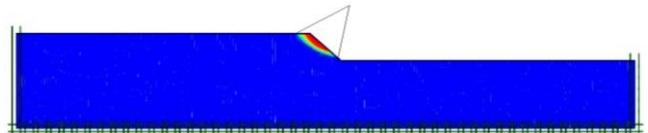


Fig. 11. Safty factor for the static slope stability conditions

### F. Stability analysis using FE

To develop a digital model EF for dynamic analysis of the incorporated slope, the dynamic modulus of the Plaxis 2D v.8.4 (Brinkgreve, 2002) was used. The basic configuration of the static model has been maintained; however, many subtle changes had to be incorporated to allow proper functionality of the dynamic model. Such modifications can be set as the calibration of the model (DEY A. et al., 2011). Several key aspects must be taken into account for such a calibration, and they are listed and discussed as follows:

- Constitutive model and material properties: For the dynamic analysis, the soil is modeled as Mohr-Coulomb material. The built slope was modeled using the previously specified properties by adding depreciation Rayleigh
- Damping settings: In most dynamic EF codes, the viscous damping is simulated by the well-known Rayleigh formulation using two coefficients,  $\alpha_R$  and  $\beta_R$  as:

$$C = \alpha_R M + \beta_R K \quad (12)$$

According to the wording of Rayleigh (used Plaxis code), modal damping  $\xi_n = \xi (\omega_n)$  depends on the natural circular frequency of  $\omega_n$  system by:

$$\xi_n = \frac{1}{2} \left( \frac{\alpha_R}{\omega_n} + \beta_R \right) \quad (13)$$

The amplification function (Roësset, 1970) for a soil layer resting on a rigid substrate (Figure 9) is:

$$A(f) = \frac{1}{\sqrt{\cos^2(2\pi \frac{H}{V_s} f) + (2\pi \frac{H D}{V_s} f)^2}} \quad (14)$$

In previous cases, the  $n^{th}$  natural frequency  $f_n$ :

$$f_n = \frac{\omega_n}{2\pi} \approx \frac{V_s}{4H} (2n - 1) \quad (15)$$

In terms of the amplification function (equation (12)):

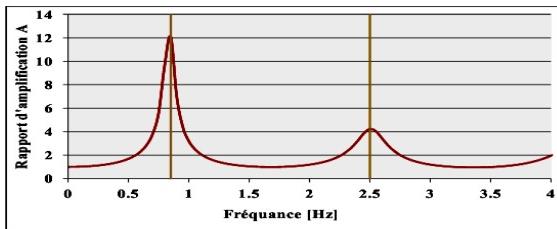


Fig. 12. Amplification function of the soil substratum.

The first frequency ( $f_1 = 0.85$  Hz); The second frequency ( $f_2 = 2.5$  Hz).

The Rayleigh damping coefficients are:  $\alpha_R = 0.398$  and  $\beta_R = 0.0047$

Using a calibration technique, leading to EF results (2D) comparable to those obtained by equivalent linear analysis of the frequency domain (Fig.13 and Fig.14).

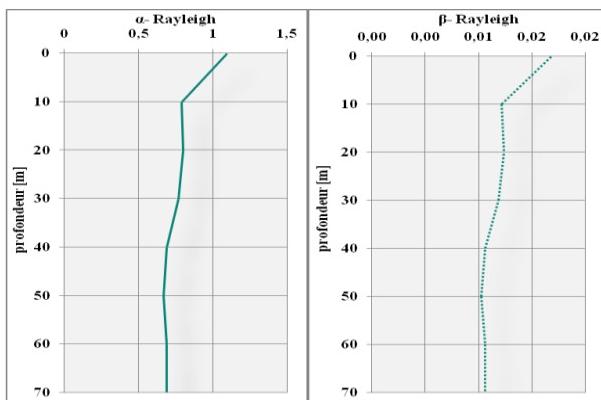


Fig. 13. The Rayleigh damping coefficients Using a calibration technique

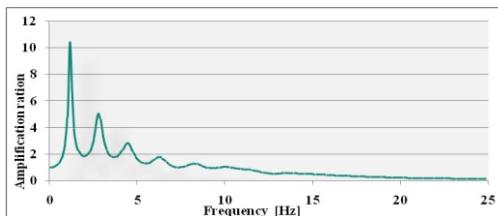


Fig. 14. Amplification function of the soil substratum.

Networking and input excitation (Fig 15): Whenever a numerical analysis is performed, the influence of the mesh must be examined. Kuhlmeyer and Lysmer (1973) suggest an element size:

$$\Delta l < \frac{\lambda}{10} = \frac{V_s}{10 f} = \frac{324.7}{8 \times 25} = 2.11 \text{ m} \quad (14)$$

Où  $f = 25$ Hz la fréquence maximale du domaine d'intérêt.

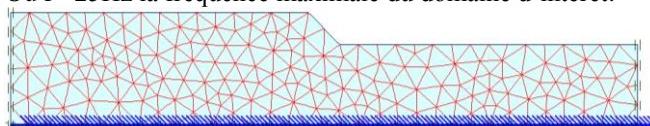


Fig. 15. Mesh of the model by triangular elements with 15 nodes.

Boundary conditions: An absorbing boundary is referred to absorb the increments of stress on the boundaries caused by dynamic loading, which would otherwise be reflected inside the soil body ( $C_1 = 1$  and  $C_2 = 0.25$ ). It can be seen that as the field width increases, the quality of spectra also improves.

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This can be interpreted due to the reduction of interference of the domain boundary on the reaction of the medium due to the reflection of the waves at the border. Therefore, it may be advisable to consider a field width of more than 10 times the size of the model to simulate reliably the actual situation.

Numerical Attenuation: The iterative process in Plaxis code for dynamic analysis is defined by the dynamic sub-steps, the two constants  $\alpha$  and  $\beta$ .

The permanent displacement: Displacement of points A and B (selected at the bottom and top of the slope) versus time seismic loading are shown in Fig.16.

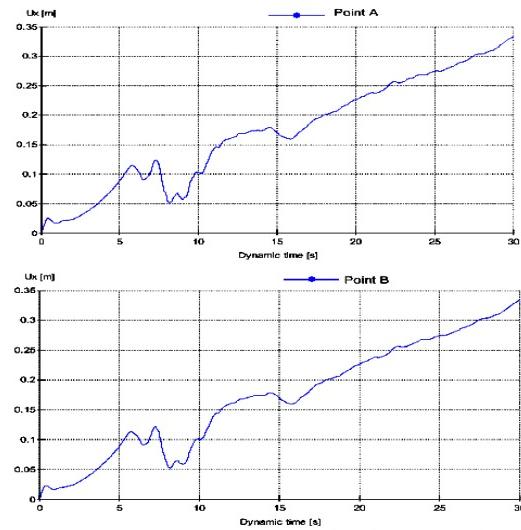


Fig. 16. Total displacements of points A and B .

#### G. Newmark permanent displacement

The first step is to calculate the seismic coefficient  $K_{crit}$  critical to determine the acceleration threshold beyond which the movement of the slope will be initiated by a permanent displacement ( $u \neq 0$ ).

According to Newmark (eq.15), the critical acceleration is equal to  $0.63 \text{ m/s}^2$  where  $\alpha = 30^\circ$  is the tilt angle to the vertical from breaking the circle, the critical seismic coefficient  $a_c = (F_S - 1) 9.81 \sin(\alpha)$  (15)

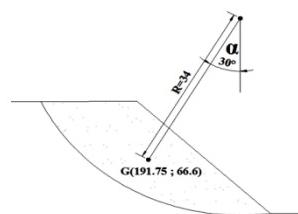


Fig. 17. Mesure of slope angle.

The portion surrounded by a circle in Fig.18 represents the range where the acceleration of the seismic record is greater than the critical acceleration, which implies the existence of a permanent displacement.

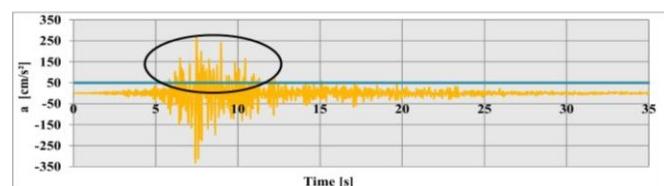


Fig. 18. The range where the acceleration of seismic recording exceed  $a_{crit}$  .

The results of permanent displacements obtained by the methods based on the concept of sliding block are listed in Table 3 and represented by the ratio  $k_c / k_m$  in Fig.19, as:  $k_m = 0.33g$ ;  $v_m = 17.89\text{cm} / \text{s}$ ;  $I_a = 0.669 \text{ m} / \text{s}$ .

TABLE III. PERMANENT DISPLACEMENTS OBTAINED BY THE METHODS BASED ON THE CONCEPT OF SLIDING BLOCK

	<b>Methodes</b>	<b>D [cm]</b>
<b>Methods using the maximum acceleration and maximum velocity:</b>	Newmark performed	10.17
	Whitman et Liao	9.52
	Other authors	26.94
<b>Methods using the maximum acceleration and intensity:</b>	Ambraseys and Menu	46.23
	Jibson (1994)	9.43
	Jibson and al., (1998)	13.30
<b>PLAXIS</b>	Point A	42.8
	Point B	40.2

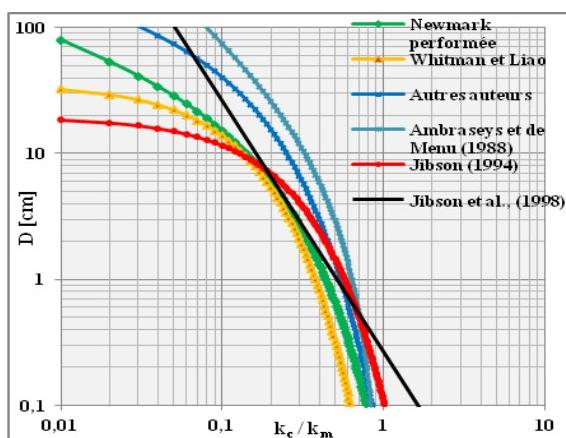


Fig. 19. Representation of permanent slope displacement.

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## V. CONCLUSION

Dynamic finite element analysis can be considered among the most comprehensive tools available in geotechnical earthquake engineering for their ability to provide information on the deformation / movement and distribution of soil stresses and forces acting on the structural elements that interact with the earth (PIANC, 2001). However, they require at least a constitutive model suitable soil, proper characterization of ground by the tests in-situ and laboratory and a proper definition of the seismic excitation. The reaction of a finite element model is also conditioned by the arrangement of several parameters influencing the sources of energy dissipation in time domain analysis. The amount of damping exhibited by a discrete digital system is determined by the choice of (after C. Visone, 2008):

- Constitutive model (materiel damping); models the effects of viscous and hysteretic dissipation of energy in the soils.
- Equations integration scheme (numerical damping); appears as a consequence of the numerical algorithm of the dynamic equilibrium of solution in the time domain.
- Boundary conditions; affect how the digital model transmits the specific energy of the stress wave outside the area.